Vibration: It is a process in which body executes Too & Fro motion with some periodic time

Ex: Bar pendulum, simple pendulum, mass attached to the spring.

Types of vibration:

① Free vibration (Natural vibration)

② Forced vibration

③ Damped vibration

① **Free vibration:** It is a vibration in which initial displacement is given, there after no external force is applied to the body. such type of vibration is called free vibration.

If the amplitude of the free vibration does not decreases then it is called as undamped free vibration.

If the amplitude of the free vibration decreases gradually then it is called damped free vibration

Ex. for Free vibration:

Simple pendulum given with initial displacement.

② **Forced vibration:** It is a vibration in which external force is applied to a body then such type of a vibration is called as forced vibration.

The vibration will have same frequency as that of applied external force.

Ex: Simple pendulum to which external periodic force is applied.

③ **Damped vibration:** It is a vibration in which amplitude decreases with time.

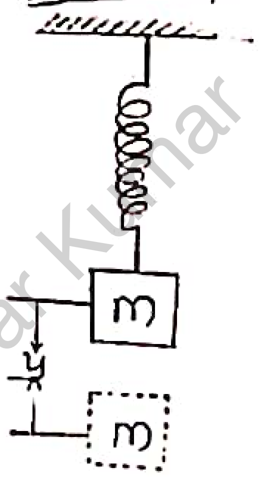
Ex: Simple pendulum under the gravitational force.

(2) **Natural frequency**: It is a frequency of vibration [OR] It is number of oscillat... per second.

Resonance: It is a phenomena in which frequency of the applied force is equal to natural frequency of the system. There will be increase in amplitude of vibra...

Ex: strike a tuning fork and hold it ... the tube containing water. If the le... of the water in the tube is gradual... decreased, the length of the air column increases. When the natural frequency ... the air column equal to the frequen... tuning fork. A very loud sound is produced. Now the air column is said to be resonance.

Undamped vibration:



Let us consider a body of mass 'm' attached to the spring of spring constant 'K'.

Let the body execute SHM, the KE of a body is

$$KE = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 \quad \text{------} \ (1)$$

& PE of a body is

$$PE = \frac{1}{2} K y^2 \quad \text{------} \ (2)$$

Then total energy of a body is

$$TE = KE + PE$$

$$TE = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} K y^2 \quad \text{------} \ (3)$$

Since body is executing SHM, the total energy is constant.

So, $TE = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} K y^2 = \text{constant}$

Now differentiate above eq^n with respe... to time 't'

③ $\frac{1}{2} m \, 2 \frac{dy}{dt} \frac{d^2y}{dt^2} + \frac{1}{2} K \, 2y \frac{dy}{dt} = 0$

$$\frac{dy}{dt}\left(m \frac{d^2y}{dt^2} + Ky\right) = 0$$

$$\therefore \quad m \frac{d^2y}{dt^2} + Ky = 0$$

Divide throughout the eqⁿ by m

$$\frac{d^2y}{dt^2} + \left(\frac{K}{m}\right)y = 0$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{......} ④$$

Where, ω = angular velocity or angular frequency

$$\therefore \quad \omega^2 = \frac{K}{m}$$

$$\omega = \sqrt{\frac{K}{m}} \quad \text{......} ⑤$$

Thus solution for the eqⁿ ④ is

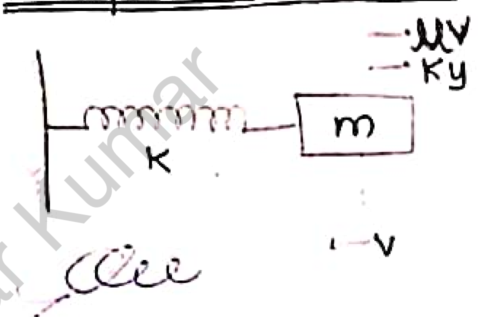
$$y = a \sin(\omega t - \alpha)$$

Eqⁿ ④ represents the eqⁿ of SHM or undamped vibration for instant of time period.

Linear frequency. $f = \frac{\omega}{2\pi}$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Damped vibration:



Let us consider a mass 'm' attached to spring with a force constant K. When external force is kept applied to the mass along the length of the spring

It starts vibrating & after some time its vibration becomes zero. because of frictional force (dissipative force) & restoring force of spring the vibration becomes zero.